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Permeability Variations in Laminar Flow through a Porous Medium Behind a Two-dimensional Grid

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Abstract: - The problem of laminar flow through a porous medium, with constant permeability, behind a two-dimensional grid is revisited in order to introduce permeability variations in the governing equations. Expressions for permeability profiles are derived when the model equations are linearized and permeability is calculated at the stagnation points of the flow and conditions on the parameters involved in the exact solution are stated. This work might be of interest in the stability analysis of flow through variable permeability media.

Keywords: - *Variable permeability, linearized Darcy-Lapwood-Brinkman equation.*

I. INTRODUCTION

The nonlinearity of the Navier-Stokes equations and the rare existence of their exact solution are also inherent in the Darcy-Lapwood-Brinkman (DLB) equation that governs the flow through porous media, or flow through mushy zones undergoing rapid freezing, [1]. The DLB equation contains the same convective inertial terms, viscous shear terms and pressure gradient as the Navier-Stokes equation, in addition to a viscous damping term that contains the permeability (*cf.* equation (2) in section 2, below). More complication is added to the process of finding exact solutions of the DLB equation when the permeability is non-constant. A variable permeability function adds one more unknown to the governing equations without adding an additional equation to render the system of governing equations determinate. This necessitates selecting a permeability function that, together with the flow variables, satisfies the governing equations, [2].

While in general finding a permeability function that satisfies the DLB equation might be formidable, the problem is simplified by using linearization techniques that are popular in finding exact solutions to the Navier-Stokes equations. An excellent review of the popular methods is provided by Wang, [3], and other methods have been introduced by various authors (*cf.* [4], [5], [6] and the references therein). An early method of solution was introduced close to a century ago by Taylor [7] who observed that in two-dimensional flow, the nonlinear convective acceleration vanishes in the Navier-Stokes equations when the vorticity is a function of the Stokes streamfunction. Kovasznay [8] obtained a linearization of the Navier-Stokes equation by using Taylor's approach and taking vorticity proportional to the streamfunction, and Lin and Tobak [9] extended Kovasznay's linearization to obtain reverse flow over a flat plate with blowing and suction.

In the study of flow through porous media, Hamdan and Ford [10] used Kovasznay's approach to study laminar flow behind a two-dimensional grid, and used the DLB equation with constant permeability. Their work, [10], illustrated the effects of constant permeability on the vorticity of the two-dimensional flow.

While flow through porous media with constant permeability has received considerable attention in the literature, [1], [11], flow problems in natural and industrial settings involve porous media with variable permeability. In addition, validity of some models of flow through porous media hinges on considerations of variable permeability in the models. In a recent article, Nield and Kuznetsov [12] demonstrated the need for, and the application of a variable permeability porous layer (used in their work as a transition layer) in analyzing flow over porous layers. Flow through the variable permeability transition layer used by Nield and Kuznetsov [12] was governed by Brinkman's equation. The DLB equation is the same in structure as the Brinkman equation except for the inertial terms [13]. These convective terms will be eliminated by linearization in this work, where the problem of laminar flow through a variable-permeability porous medium behind a two-dimensional grid is considered. This is the same problem considered by Hamdan and Ford, [10], except that the current problem involves a permeability function that must be determined so that the vorticity equation is satisfied. An extension to the approach followed in [10] will be used in the current work, and derivations of equations that the permeability functions must satisfy are obtained, then solved. Analysis carried out in this work might be of utility in stability analysis of flow through porous media with variable permeability.

II. GOVERNING EQUATIONS

Consider the flow through a porous medium of the type where the Darcy-Lapwood-Brinkman (DLB) equation is valid; that is, one in which viscous shear and macroscopic inertia are important. The flow of an incompressible fluid through the medium is governed by the equations of continuity and momentum, given by, [13]:

$$\nabla \bullet \vec{v} = 0 \tag{1}$$

$$\rho(\vec{v} \bullet \nabla)\vec{v} = -\nabla p + \mu^* \nabla^2 \vec{v} - \frac{\mu}{k}\vec{v} \tag{2}$$

where $\vec{v}$ is the velocity vector field, p is the pressure, ρ is the fluid density, μ is the viscosity of the base fluid, μ^* is the effective viscosity of the fluid as it occupies the porous medium, k is the permeability (considered here a scalar function of position), ∇ is the gradient operator and ∇^2 is the laplacian. In the absence of definite information about the relationship between μ^* and μ, we will take $\mu^* = \mu$ in this work.

Considering the flow in two space dimensions, *x* and *y*, we take $\vec{v} = (u, v)$, $k = k(x, y)$ and $p = p(x, y)$. The equation of continuity can be written as:

$$u_x + v_y = 0 \tag{3}$$

and momentum equations can be expressed in the *x*- and *y*-directions, respectively, as:

$$\rho([u + u_0]u_x + vu_y) = -P_x + \mu\nabla^2 u - \frac{\mu}{k}u \tag{4}$$

$$\rho([u + u_0]v_x + vv_y) = -P_y + \mu\nabla^2 v - \frac{\mu}{k}v \tag{5}$$

where u_0 is the average velocity in the x-direction.

The governing equations (3), (4), and (5) are conveniently expressed in vorticity-velocity form as follows. Letting ξ be the vorticity of the flow, defined as:

$$\xi = \nabla \times \vec{v} = v_x - u_y \tag{6}$$

and eliminating the pressure term from equations (4) and (5) by differentiating (4) with respect to *y* and differentiating (5) with respect to *x*, then subtracting, gives:

$$[u + u_0]\xi_x + v\xi_y = \upsilon\nabla^2\xi - \frac{\upsilon}{k}\xi + \upsilon v\frac{k_x}{k^2} - \upsilon u\frac{k_y}{k^2} \tag{7}$$

where $\upsilon = \dfrac{\mu}{\rho}$ is the kinematic viscosity.

Equations (3), (6) and (7) represent three equations to be solved for the unknowns u, v and ξ. The permeability distribution is yet to be determined. These equations can be rendered dimensionless with respect to a characteristic length L that represents grid spacing, and a characteristic velocity (the average velocity u_0) by defining:

$$U = u/u_0, V = v/u_0, \Omega = \xi L/u_0, X = x/L, Y = y/L, K = k/L^2, \mathrm{Re} = \frac{Lu_0}{\upsilon} \tag{8}$$

where Re is Reynolds number. Equations (3), (6) and (7) thus take the following dimensionless forms, respectively:

$$U_X + V_Y = 0 \tag{9}$$

$$\Omega = V_X - U_Y \tag{10}$$

$$[U + 1]\,\mathrm{Re}\frac{\partial\Omega}{\partial X} + \mathrm{Re}\,V\frac{\partial\Omega}{\partial Y} = \nabla^2\Omega - \frac{\Omega}{K} + V\frac{\partial K}{\partial X}\frac{1}{K^2} - U\frac{\partial K}{\partial Y}\frac{1}{K^2} \tag{11}$$

It is thus required to solve equations (9), (10) and (11) for the unknowns U, V and Ω, for a given dimensionless permeability distribution.

III. SOLUTION METHODOLOGY

Hamdan and Ford [10] and the references therein discussed the following ways to linearize the vorticity equation when permeability is constant:

(1) If inertial terms are small compared with viscous effects, then Reynolds number is small and the vorticity equation is reduced to a linear equation.
(2) If changes in velocity are small compared with the average velocity, the quadratic terms may be neglected.
Due to variations of permeability, as can be seen from equation (11), the second linearization approach is followed in this work. Assuming that the changes in velocity are small compared with the average velocity, we neglect the quadratic terms and set:

$$U\frac{\partial\Omega}{\partial X}+V\frac{\partial\Omega}{\partial Y}=0 \tag{12}$$

Equation (11) is then reduced to:

$$\mathrm{Re}\frac{\partial\Omega}{\partial X}=\nabla^2\Omega-\frac{\Omega}{K}+V\frac{\partial K}{\partial X}\frac{1}{K^2}-U\frac{\partial K}{\partial Y}\frac{1}{K^2} \tag{13}$$

Now, if the changes in velocity are not small, then we attempt to find solutions for which the quadratic terms vanish, and (12) holds.
In order to facilitate the solution to equations (9), (10) and (13), we introduce the dimensionless streamfunction of the flow, $\Psi(X,Y)$, the existence of which is implied by the dimensionless equation of continuity (9), and defined in terms of the dimensionless velocity components by:

$$U=\Psi_Y \tag{14}$$

and

$$V=-\Psi_X \tag{15}$$

Using (14) and (15) in (10), the following streamfunction equation is obtained:

$$\nabla^2\Psi=-\Omega \tag{16}$$

The equations to be solved are therefore the streamfunction equation (16) and the vorticity equation (13). Velocity components can then be evaluated from (15) and (16).
Assuming that the streamfunction is of the periodic form:

$$\Psi=F(X)\sin(2\pi Y) \tag{17}$$

where $F(X)$ is to be determined, then the velocity components and vorticity take the following forms, respectively:

$$U=\Psi_Y=2\pi F(X)\cos(2\pi Y) \tag{18}$$

$$V=-\Psi_X=-F'(X)\sin(2\pi Y) \tag{19}$$

$$\Omega=[4\pi^2F(X)-F''(X)]\sin(2\pi Y) \tag{20}$$

Substituting (18), (19) and (20) in (12) yields:

$$[F'(X)F''(X)-F(X)F'''(X)]\pi\sin(4\pi Y)=0 \tag{21}$$

Equation (21) is satisfied when $Y=\frac{n}{4};n=0,1,2,3,...$, or when:

$$F'(X)F''(X)-F(X)F'''(X)=0 \tag{22}$$

Equation (22) can be written in the form:

$$\frac{F'(X)}{F(X)}=\frac{F'''(X)}{F''(X)} \tag{23}$$

which, upon integrating once and simplifying, gives:

$$cF''(X)=F(X) \tag{24}$$

where c is an arbitrary constant.
Equation (24) is satisfied by $F(X)$ of the form:

$$F(X)=Ae^{\alpha X} \tag{25}$$

where A and α are constants.
Using (25) in (17) gives:

$$\Psi=Ae^{\alpha X}\sin(2\pi Y) \tag{26}$$

and using (26) in (18), (19) and (20) yields, respectively:

$$U=2\pi Ae^{\alpha X}\cos(2\pi Y) \tag{27}$$

$$V = -\alpha A e^{\alpha X} \sin(2\pi Y) \tag{28}$$

$$\Omega = [4\pi^2 - \alpha^2] A e^{\alpha X} \sin(2\pi Y) = [4\pi^2 - \alpha^2]\Psi \tag{29}$$

For the case of flow through porous media with constant permeability, [10], it was argued that equation (29) gives the vorticity in terms of the streamfunction, by virtue of its definition, and is not based on solving the vorticity equation (equation (13) with constant permeability). The choice of $F(X)$ was based on the satisfaction of (12), while the solution for vorticity must satisfy both (12) and (13). Hence, the form of vorticity is determined by (29), namely:

$$\Omega = G(X)\sin(2\pi Y) \tag{30}$$

where $G(X)$ is a function to be determined by substituting (30) in (13) when permeability is constant.

Since in the current work the permeability is a variable function of position, the above process of solving (13) while implementing (30) is cumbersome. We modify the above approach by taking (29) to represent the solution for vorticity and find the permeability distribution that satisfies (13) when vorticity is given by (29).

In order to determine the permeability distribution that guarantees that the vorticity given by (29) satisfies the vorticity equation (13), equations (27), (28) and (29) are substituted in (13) to get:

$$\mathrm{Re}\,\alpha + [4\pi^2 - \alpha^2] = -\frac{1}{K} - \frac{\alpha}{[4\pi^2 - \alpha^2]}\frac{\partial K}{\partial X}\frac{1}{K^2} - \frac{2\pi\cot(2\pi Y)}{[4\pi^2 - \alpha^2]}\frac{\partial K}{\partial Y}\frac{1}{K^2} \tag{31}$$

Equation (31) is solved for the cases of $K = K(X)$ only, and $K = K(Y)$ only.

Case 1: $K = K(X)$

Equation (31) in this case reduces to

$$\mathrm{Re}\,\alpha + [4\pi^2 - \alpha^2] = \left(-\frac{1}{K}\right) - \frac{\alpha}{[4\pi^2 - \alpha^2]}\frac{d}{dX}\left(\frac{-1}{K}\right) \tag{32}$$

which is a linear, first order ordinary differential equation in the unknown $\frac{-1}{K}$. Integration factor, *I.F.*, is given by:

$$I.F. = \exp\left\{-\frac{4\pi^2 - \alpha^2}{\alpha} X\right\} \tag{33}$$

and solution given by

$$K = \frac{-1}{\alpha\,\mathrm{Re} + 4\pi^2 - \alpha^2 + C_1 \exp\left\{\frac{4\pi^2 - \alpha^2}{\alpha} X\right\}} \tag{34}$$

where C_1 is an arbitrary constant.

Case 2: $K = K(Y)$

Equation (31) in this case reduces to

$$\mathrm{Re}\,\alpha + [4\pi^2 - \alpha^2] = \left(-\frac{1}{K}\right) - \frac{2\pi\cot(2\pi Y)}{[4\pi^2 - \alpha^2]}\frac{d}{dY}\left(\frac{-1}{K}\right) \tag{35}$$

which is a linear, first order ordinary differential equation in the unknown $\frac{-1}{K}$. Integration factor, *I.F.*, is given by:

$$I.F. = \exp\left[\frac{4\pi^2 - \alpha^2}{4\pi^2} \ln\left|\cos(2\pi Y)\right|\right] \tag{36}$$

and solution given by

$$K = \frac{-1}{\alpha\,\mathrm{Re} + [4\pi^2 - \alpha^2] + C_2 \exp\left[-\frac{4\pi^2 - \alpha^2}{4\pi^2} \ln\left|\cos(2\pi Y)\right|\right]} \tag{37}$$

where C_2 is an arbitrary constant and $Y \neq \frac{n}{4}; n = 1,2,3,\ldots$.

IV. Results and Discussion

IV.1. Total Flow

Equations (26)-(28) represent the solutions for Ψ, U, V and Ω. The corresponding quantities for the total flow are given by:

$$U + 1 = 1 + 2\pi A e^{\alpha X} \cos(2\pi Y) \tag{38}$$

$$V = -\alpha A e^{\alpha X} \sin(2\pi Y) \tag{39}$$

$$\Psi^* = Y + A e^{\alpha X} \sin(2\pi Y) \tag{40}$$

$$\Omega = [4\pi^2 - \alpha^2] A e^{\alpha X} \sin(2\pi Y) \tag{41}$$

where $\Psi^* (= \Psi + \Psi_0)$ is the total streamfunction such that $\frac{\partial \Psi^*}{\partial Y} = U + 1$, and Ψ_0 is the perturbing stream.

IV.2. Stagnation Points

Stagnation points of the total flow occur where $V = 0$ and $U + 1 = 0$. Locations of the stagnation points are at $(X, Y) = (0, \frac{n}{2}); n = 0,1,2,3,...$, as explained as follows. Taking $V = 0$ in (39), and noting that $\alpha \neq 0$ and $A \neq 0$, then $\sin(2\pi Y) = 0$, $Y = \frac{n}{2}$; $n = 0,1,2,3...$. Taking $U + 1 = 0$ in (38) gives $A = \frac{-1}{2\pi e^{\alpha X} \cos(2\pi Y)}$. Assuming that stagnation occurs at $X = 0$ when $Y = \frac{n}{2}$, gives $A = \frac{-1}{2\pi \cos(n\pi)}$. Now, if the zero streamline passes through the stagnation point, then $\Psi^* = 0$ when $Y = 0$ (that is, when $n = 0$), then $A = \frac{-1}{2\pi}$. Taking $A = \frac{-1}{2\pi}$ in (38)-(41), gives:

$$U + 1 = 1 - e^{\alpha X} \cos(2\pi Y) \tag{42}$$

$$V = \frac{\alpha}{2\pi} e^{\alpha X} \sin(2\pi Y) \tag{43}$$

$$\Psi^* = Y - \frac{1}{2\pi} e^{\alpha X} \sin(2\pi Y) \tag{44}$$

$$\Omega = -[2\pi - \frac{\alpha^2}{2\pi}] e^{\alpha X} \sin(2\pi Y) \tag{45}$$

IV.3. Determination of Constants

The value $A = \frac{-1}{2\pi}$, which appears in the total flow quantities, was determined in the previous subsection through consideration of stagnation points. It is the same value obtained in [10] for the case flow through a porous medium of constant permeability since the locations of stagnation points are the same for the problem at hand for the cases of constant and variable permeability by virtue of the fact that the streamfunction and velocity components in the total flow are the same.

In the assumption of the form of vorticity function, and in determination in the case of constant permeability, the value of the constant α was tied to the roots of the auxiliary equation, [10], of the governing ordinary differential equation for $G(X)$ of equation (30). For the current problem where permeability is non-constant, finding a value for α may not be as easy, however, there are some restrictions that can be stated as follows:

(a) In order to have a non-zero vorticity in the flow field, equation (45) suggests that $\alpha \neq 2\pi$.

(b) The permeability functions, equations (34) and (37), tie in Reynolds number, α and C_1 (or C_2). Their values must be chosen such that the permeability at any point in the flow field is positive, hence the following must hold:

$$\alpha \operatorname{Re} + 4\pi^2 - \alpha^2 + C_1 \exp\{\frac{4\pi^2 - \alpha^2}{\alpha} X\} < 0 \tag{46}$$

$$\alpha \operatorname{Re} + [4\pi^2 - \alpha^2] + C_2 \exp[-\frac{4\pi^2 - \alpha^2}{4\pi^2} \ln \left|\cos(2\pi Y)\right|] < 0 \tag{47}$$

For instance, if $\operatorname{Re} = 0$ then at the stagnation points $(X, Y) = (0, \frac{n}{2})$; $n = 0,1,2,3,...$, we must have:

$$C_1 < \alpha^2 - 4\pi^2 \tag{48}$$

$$C_2 < \alpha^2 - 4\pi^2 . \tag{49}$$

IV. Comparison with the Case of Constant Permeability and Kovasnay's Solution

Kovasnay's approach [8] for Navier-Stokes flow was followed both in this work and in the work of Hamdan and Ford [10] for flow through constant permeability media. Subsequently, the form of the streamfunction (hence the velocity components) is the same in all three studies. Differences occur in the vorticity and permeability.

Kovasnay [8] obtained the following vorticity expression for the Navier-Stokes flow:

$$\Omega = \frac{\operatorname{Re} m}{2\pi} e^{mX} \sin(2\pi Y) \tag{50}$$

where

$$m = \frac{\operatorname{Re}}{2} \mp \sqrt{\frac{(\operatorname{Re})^2}{2} + 4\pi^2} \tag{51}$$

Hamdan and Ford [10] obtained the following vorticity expression for the DLB case with constant permeability:

$$\Omega = [\frac{1}{2\pi K} + \frac{\operatorname{Re} m}{2\pi}] e^{mX} \sin(2\pi Y) \tag{52}$$

where m is as given in (51). Equation (52) demonstrates the effect of the constant permeability on the flow, and how it enters the vorticity equation.

The current work calculates the vorticity through its definition in terms of the streamfunction, as given in (45), and takes into account the effects of the permeability function and Reynolds number through equations (34) and (37), wherein Reynolds number enters the definition of the permeability and influences the values of permeability without the need to guess a suitable range of Re.

$$\alpha \operatorname{Re} + 4\pi^2 - \alpha^2 + C_1 \exp\{\frac{4\pi^2 - \alpha^2}{\alpha} X\} < 0 \tag{46}$$

$$\alpha \operatorname{Re} + [4\pi^2 - \alpha^2] + C_2 \exp[-\frac{4\pi^2 - \alpha^2}{4\pi^2} \ln \left|\cos(2\pi Y)\right|] < 0 \tag{47}$$

C_1	C_2	Re	Condition on α
0	0	0	$\alpha \in (-\infty, -2\pi) \cup (2\pi, +\infty)$
0	0	1	$\alpha \in (-\infty, \frac{1 - \sqrt{1 + 16\pi^2}}{2}) \cup (\frac{1 + \sqrt{1 + 16\pi^2}}{2}, \infty)$
0	0	Re	$\alpha \in (-\infty, \frac{\operatorname{Re} - \sqrt{\operatorname{Re}^2 + 16\pi^2}}{2}) \cup (\frac{\operatorname{Re} + \sqrt{\operatorname{Re}^2 + 16\pi^2}}{2}, +\infty)$
r	r	Re	$\alpha \in (-\infty, \frac{\operatorname{Re} - \sqrt{\operatorname{Re}^2 + 4[4\pi^2 + r]^2}}{2}) \cup (\frac{\operatorname{Re} + \sqrt{\operatorname{Re}^2 + 4[4\pi^2 + r]^2}}{2}, +\infty)$
r_1	r_2	Re	*When K=K(X):* $\alpha \in (-\infty, \frac{\operatorname{Re} - \sqrt{\operatorname{Re}^2 + 4[4\pi^2 + r_1]^2}}{2}) \cup (\frac{\operatorname{Re} + \sqrt{\operatorname{Re}^2 + 4[4\pi^2 + r_1]^2}}{2}, +\infty)$ *When K=K(Y):*

			$\alpha \in \left(-\infty, \frac{Re - \sqrt{Re^2 + 4[4\pi^2 + r_2]^2}}{2}\right) \cup \left(\frac{Re + \sqrt{Re^2 + 4[4\pi^2 + r_2]^2}}{2}, +\infty\right)$

Table 1. Ranges of Values of α at the Stagnation Points

IV. Graphical Representation of Solutions

In what follows we present the solutions for cases 1 and 2 graphically, using two- and three-dimensional plots for permeability, streamfunction and vorticity over a sub-region of the flow domain.

For Case 1:

Equation (37) can be shown in the following two-dimensional figures for the values of C_2 = 1, 2, 5, 10 and 20, and Re = 0, 1, and 10. The values of α used are in accordance with **Table** 2.

Re	$\alpha = 10 + \frac{Re + \sqrt{Re^2 + 4[4\pi^2 + 1]^2}}{2}$
0	$\alpha = 4\pi^2 + 11$
1	$\alpha = \frac{21}{2} + \frac{1}{2}\sqrt{1 + 4(4\pi^2 + 1)^2}$
10	$\alpha = 15 + \sqrt{25 + 4(4\pi^2 + 1)^2}$

Table 2. Choice of α used in Case 1.

For the range of Re tested, permeability changes are very small between stagnation points (at Y=0.5 and Y=1) and a noticeable increase takes place as we get closer to the stagnation point. The amount of increase in permeability is greater for greater values of C_2, as shown in Fig. 1.1-1.5. A typical three-dimensional graph of $K(Y)$ is given in Fig. 1.6 which shows the permeability distribution as we approach the stagnation point at Y=1.

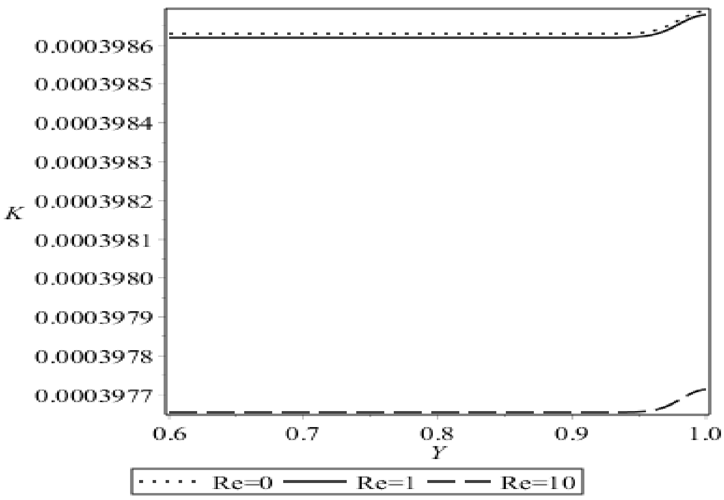

Fig.1.1. $K(Y)$ **when** $C_2 = 1$

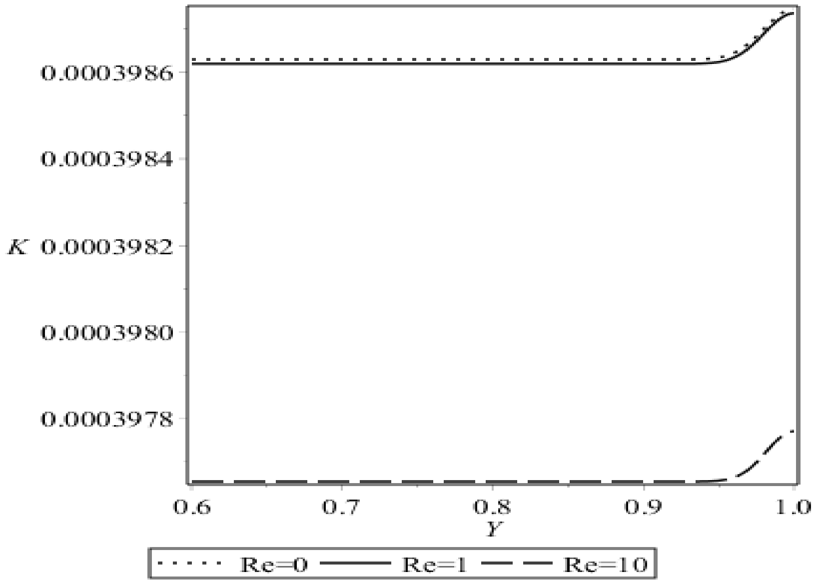

Fig1.2. $K(Y)$ **when** $C_2 = 2$

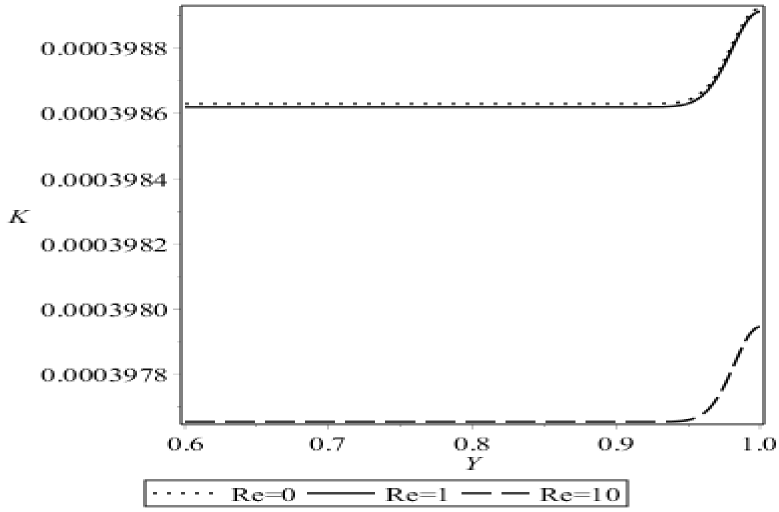

Fig 1.3. $K(Y)$ **when** $C_2 = 5$

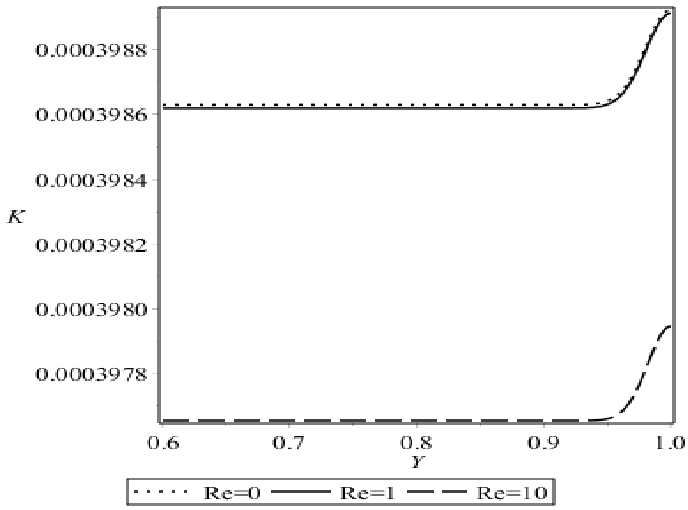

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Fig 1.4. $K(Y)$ **when** $C_2 = 10$

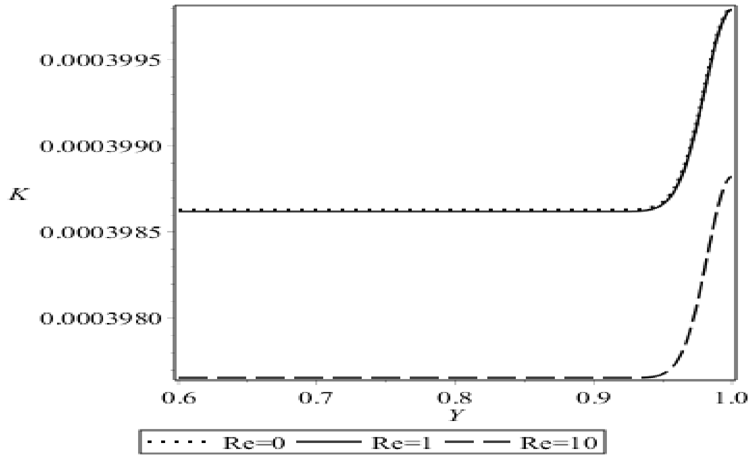

Fig 1.5. $K(Y)$ **when** $C_2 = 20$

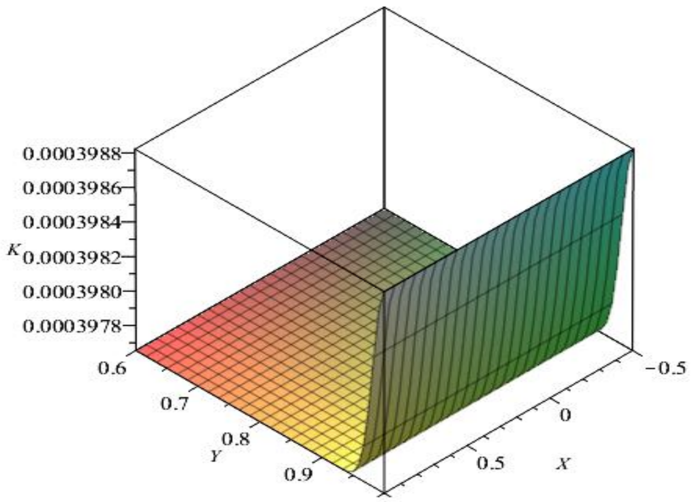

Fig 1.6. $K(Y)$ **when** $C_2 = 20$ and $\mathrm{Re} = 10$

For Case 2:

Equation (34) is shown graphically in the following Figs. 2.1 and 2.2. While small changes in the permeability distribution $K(X)$ take place as we increase Re, a noticeable increase takes place as we get closer to the stagnation point at $X = 0$. The choice of α in this case is determined using **Table** 3.

Re	$\alpha = 10 + \dfrac{\mathrm{Re} + \sqrt{\mathrm{Re}^2 + 4[4\pi^2 + 4]^2}}{2}$
0	$\alpha = 4\pi^2 + 14$
1	$\alpha = \dfrac{21}{2} + \dfrac{1}{2}\sqrt{1 + 4(4\pi^2 + 4)^2}$
10	$\alpha = 15 + \sqrt{25 + 4(4\pi^2 + 4)^2}$

Table 3. Choice of α used in Case 2.

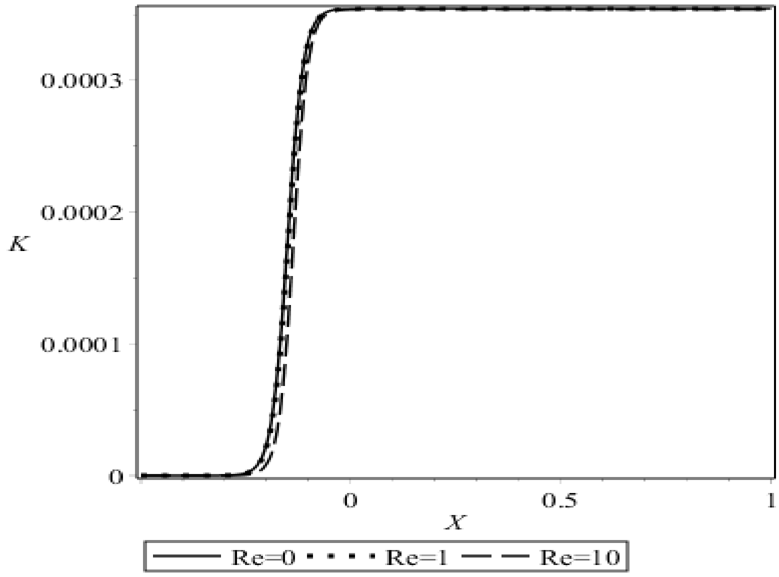

Fig 2.1. $K(X)$ **when** $C_1 = -1$

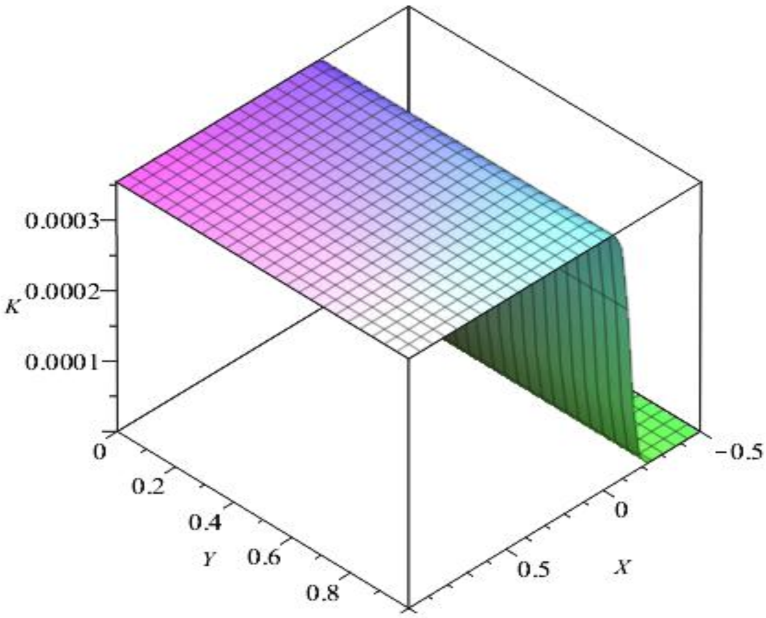

Fig 2.2. $K(X)$ **when** $C_1 = -1$ and $\text{Re} = 10$

The streamfunction distribution is given by equation (44). Streamsurfaces are shown in **Figs.** 3.1-3.4 for different values of α . The values of α are selected here are for illustration purposes and show regions of changes in streamfunction. Changes in α result in changes in the location of the maximum and minimum locations as we approach stagnation points.

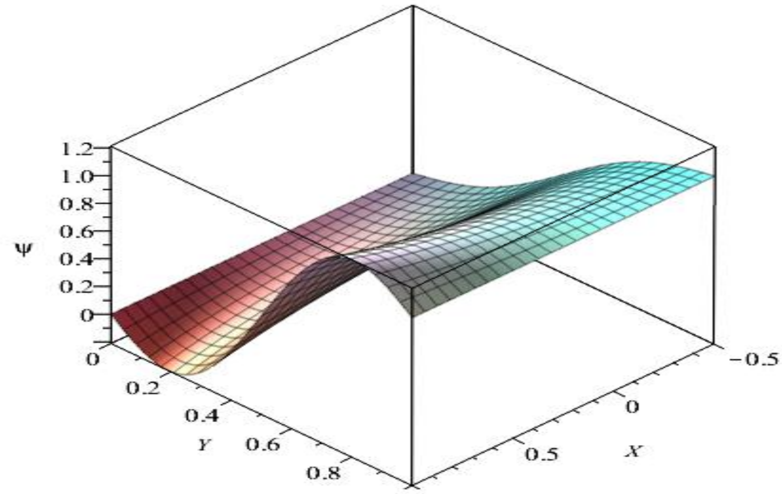

Fig 3.1. Streamsurface when $\alpha = 1$

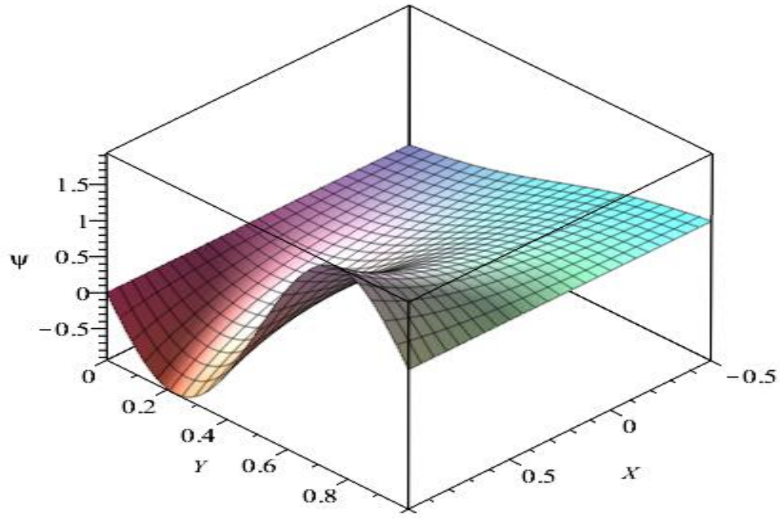

Fig 3.2. Streamsurface when $\alpha = 2$

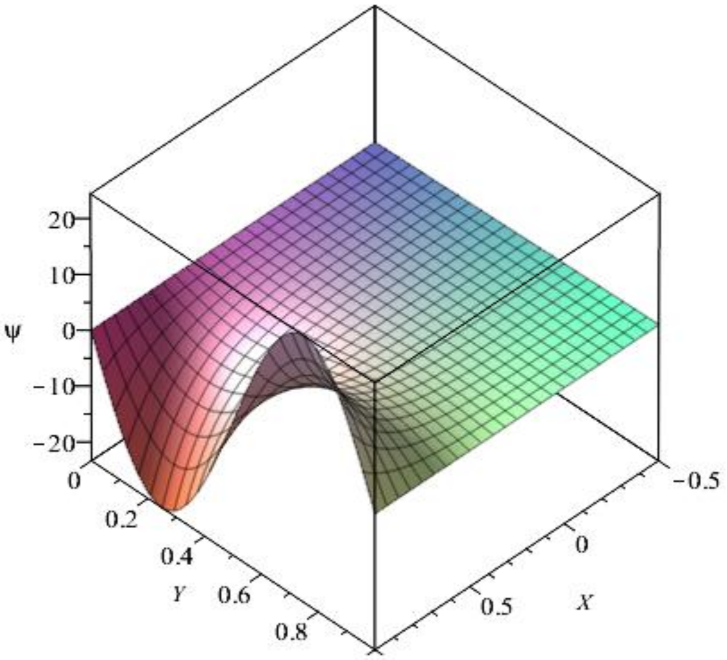

Fig 3.3. Streamsurface when $\alpha = 5$

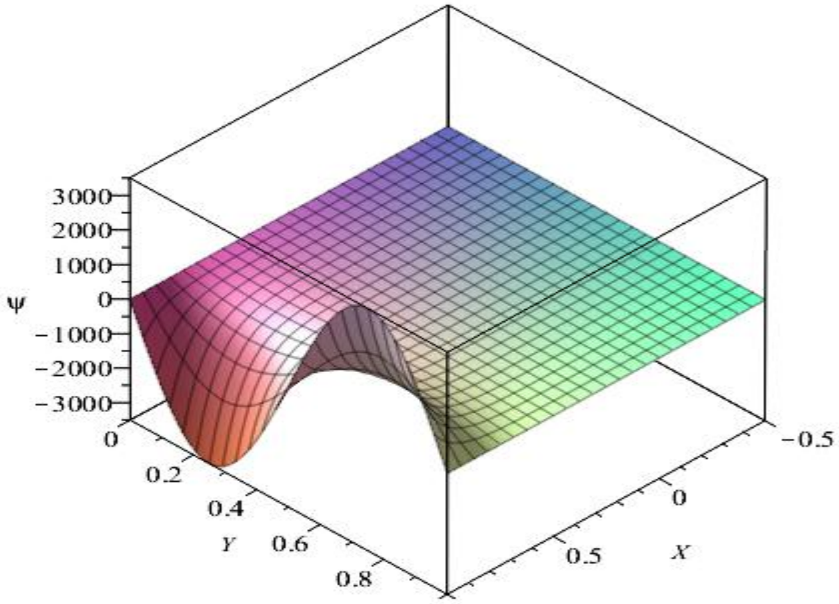

Fig 3.4. Streamsurface when $\alpha = 10$

Vorticity distribution, (45), is shown in **Figs.** 4.1-4.4 for different values of α . The values of α are selected here for illustration purposes and show the significant changes in vorticity. Changes in α result in changes in the location of the maximum and minimum locations as we approach stagnation points.

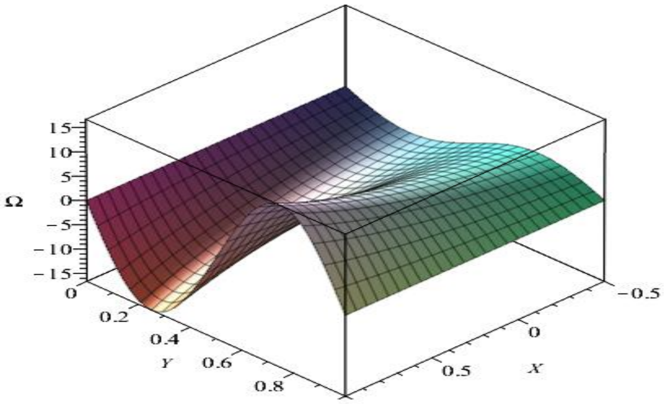

Fig 4.1. Vorticity distribution when $\alpha = 1$

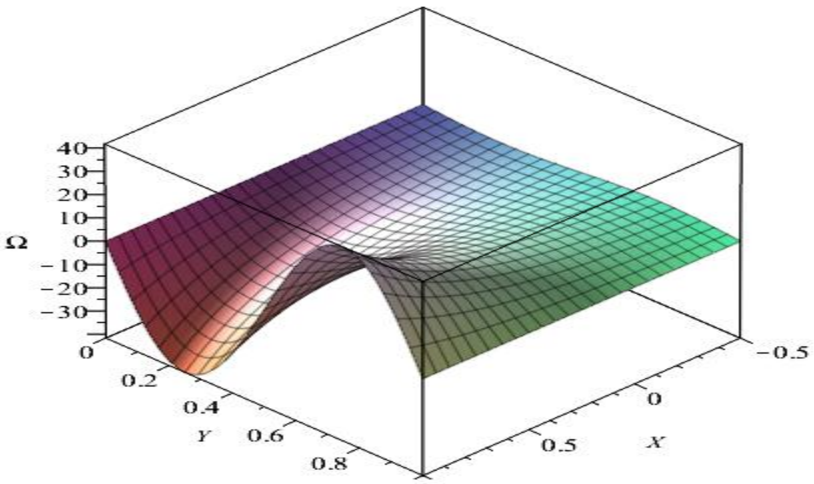

Fig 4.2. Vorticity when $\alpha = 2$

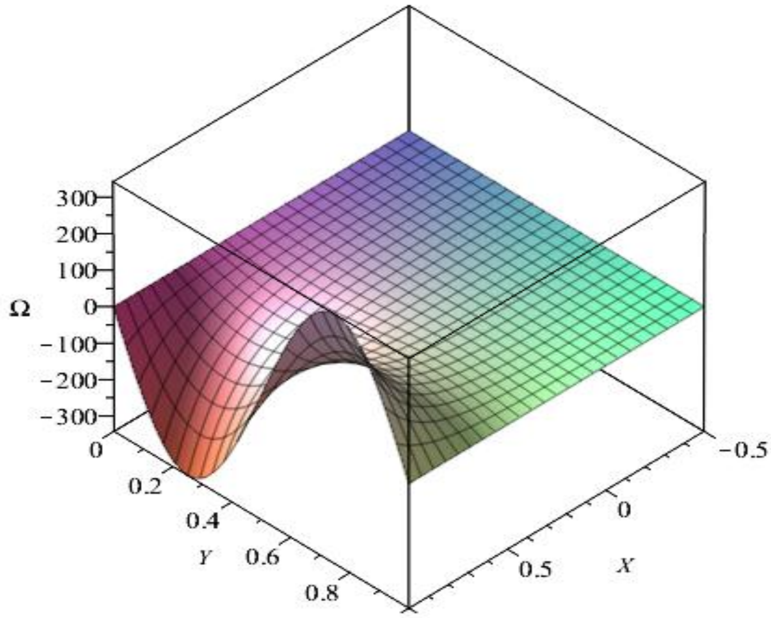

Fig 4.3. Vorticity when $\alpha = 5$

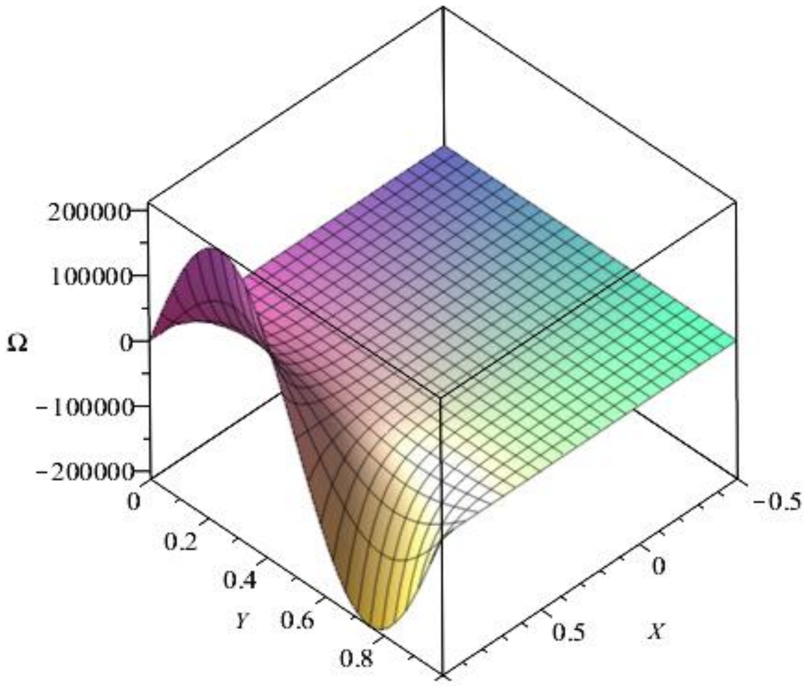

Fig 4.4. Vorticity when $\alpha = 10$

V. CONCLUSION

The problem of flow through a variable-permeability porous structure behind a two-dimensional grid was considered in this work in order to determine the permeability functions under which the streamfunction is periodic in *Y*. A method of linearization bases on Kovasznay's approach has been followed in this work, and solutions have been found with the assumptions that $Y \neq \frac{n}{4}; n = 1,2,3,...$. and $\alpha \neq 2\pi$. Conditions on the arbitrary constants that are involved in the permeability functions have been stated in (46) and (47). Stagnation points of the flow occur at

$(X, Y) = (0, \frac{n}{2}); n = 0,1,2,3,...$.

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